1985/1. Find all integer solutions to: $a + b + c = 24$, $a^2 + b^2 + c^2 = 210$, $abc = 440$.

1985/2. P is a point inside the equilateral triangle ABC such that PA = 5, PB = 7, PC = 8. Find AB.

1985/3. Find the roots r_1, r_2, r_3, r_4 of the equation $4x^4 - ax^3 + bx^2 - cx + 5 = 0$, given that they are positive reals satisfying $r_1/2 + r_2/4 + r_3/5 + r_4/8 = 1$.

1985/4. The reals x, y, z satisfy $x \neq 1$, $y \neq 1$, $x \neq y$, and $(yz - x^2)/(1 - x) = (xz - y^2)/(1 - y)$. Show that $(yx - x^2)/(1 - x) = x + y + z$.

1985/5. The function f(n) is defined on the positive integers and takes non-negative integer values. It satisfies (1) f(mn) = f(m) + f(n), (2) f(n) = 0 if the last digit of n is 3, (3) f(10) = 0. Find f(1985).

1985/6. O is the circumcenter of the triangle ABC. The lines AO, BO, CO meet the opposite sides at D, E, F respectively. Show that 1/AD + 1/BE + 1/CF = 2/AO.

1987/1. Find f(x) such that $f(x)^2 f((1-x)/(1+x)) = 64x$ for x not 0, ±1.

1987/2. In the triangle ABC, the midpoints of AC and AB are M and N respectively. BM and CN meet at P. Show that if it is possible to inscribe a circle in the quadrilateral AMPN (touching every side), then ABC is isosceles.

1987/3. Show that if $(2 + \sqrt{3})^k = 1 + m + n\sqrt{3}$, for positive integers m, n, k with k odd, then m is a perfect square.

1987/4. Define the sequence $p_1, p_2, p_3, \ldots$ as follows. $p_1 = 2$, and p_n is the largest prime divisor of $p_1 p_2 \ldots p_{n-1} + 1$. Prove that 5 does not occur in the sequence.

1987/5. Show that the roots r, s, t of the equation $x(x - 2)(3x - 7) = 2$ are real and positive. Find $\tan^{-1}r + \tan^{-1}s + \tan^{-1}t$.

1987/6. ABCD is a convex quadrilateral. P, Q are points on the sides AD, BC respectively such that AP/PD = BQ/QC = AB/CD. Show that the angle between the lines PQ and AB equals the angle between the lines PQ and CD.

1988/1. The sides of a triangle form an arithmetic progression. The altitudes also form an arithmetic progression. Show that the triangle must be equilateral.

1988/2. The positive integers a, b, c, d, p, q satisfy $ad - bc = 1$ and $a/b > p/q > c/d$. Show that $q \geq b + d$ and that if $q = b + d$, then $p = a + c$.

1988/3. P is a fixed point in the plane. Show that amongst triangles ABC such that PA = 3, PB = 5, PC = 7, those with the largest perimeter have P as incenter.

1988/4. Points $A_1, A_2, \ldots , A_n$ are equally spaced on the side BC of the triangle ABC (so that $BA_1 = A_1A_2 = \ldots = A_{n-1}A_n = A_nC$). Similarly, points $B_1, B_2, \ldots , B_n$ are equally spaced on the

side CA, and points $C_1, C_2, \ldots , C_n$ are equally spaced on the side AB. Show that $(AA_1^2 + AA_2^2 + \ldots + AA_n^2 + BB_1^2 + BB_2^2 + \ldots + BB_n^2 + C_1^2 + \ldots + CC_n^2)$ is a rational multiple of $(AB^2 + BC^2 + CA^2)$.

1988/5. Let $k^3 = 2$ and let x, y, z be any rational numbers such that $x + y k + z k^2$ is non-zero. Show that there are rational numbers u, v, w such that $(x + y k + z k^2)(u + v k + w k^2) = 1$.

1988/6. Let S be the collection of all sets of n distinct positive integers, with no three in arithmetic progression. Show that there is a member of S which has the largest sum of the inverses of its elements (you do not have to find it or to show that it is unique).

1989/1. Find all real solutions to: $x + y - z = -1$; $x^2 - y^2 + z^2 = 1$, $-x^3 + y^3 + z^3 = -1$.

1989/2. Given positive real numbers x, y, z each less than $\pi/2$, show that $\pi/2 + 2 \sin x \cos y + 2 \sin y \cos z > \sin 2x + \sin 2y + \sin 2z$.

1989/3. If a, b, c, are the sides of a triangle, show that $(a - b)/(a + b) + (b - c)/(b + c) + (c - a)/(a + c) < 1/16$.

1989/4. The incircle of the triangle ABC touches AC at M and BC at N and has center O. AO meets MN at P and BO meets MN at Q. Show that MP·OA = BC·OQ.

1989/5. The function f on the positive integers satisfies $f(1) = 1$, $f(2n + 1) = f(2n) + 1$ and $f(2n) = 3 f(n)$. Find the set of all m such that m = f(n) for some n.

1989/6. Show that there are infinitely many solutions in positive integers to $2a^2 - 3a + 1= 3b^2 + b$.

1990/1. The function f is defined on the non-negative integers. $f(2^n - 1) = 0$ for n = 0, 1, 2, ... If m is not of the form $2^n - 1$, then $f(m) = f(m+1) + 1$. Show that $f(n) + n = 2^k - 1$ for some k, and find $f(2^{1990})$.

1990/2. I is the incenter of the triangle ABC and the incircle touches BC, CA, AB at D, E, F respectively. AD meets the incircle again at P. M is the midpoint of EF. Show that PMID is cyclic (or the points are collinear).

1990/3. $f(x) = (x + b)^2 + c$, where b and c are integers. If the prime p divides c, but p^2 does not divide c, show that f(n) is not divisible by p^2 for any integer n. If an odd prime q does not divide c, but divides f(n) for some n, show that for any r, we can find N such that q^r divides f(N).

1990/4. The circle C has diameter AB. The tangent at B is T. For each point M (not equal to A) on C there is a circle C' which touches T and touches C at M. Find the point at which C' touches T and find the locus of the center of C' as M varies. Show that there is a circle orthogonal to all the circles C'.

1990/5. A and B are opposite corners of an n x n board, divided into n^2 squares by lines parallel to the sides. In each square the diagonal parallel to AB is drawn, so that the board is divided into $2n^2$ small triangles. The board has $(n + 1)^2$ nodes and large number of line segments, each of length 1 or √2. A piece moves from A to B along the line segments. It never moves along the same segment twice and its path includes exactly two sides of every small triangle on the board. For which n is this possible?

1990/6. f(x) is a polynomial of degree 3 with rational coefficients. If its graph touches the x-axis, show that it has three rational roots.

1991/1. The number 1 or the number -1 is assigned to each vertex of a cube. Then each face is given the product of its four vertices. What are the possible totals for the resulting 14 numbers?

1991/2. Two perpendicular lines divide a square into four parts, three of which have area 1. Show that the fourth part also has area 1.

1991/3. f is a function defined on all reals in the interval [0, 1] and satisfies f(0) = 0, f(x/3) = f(x)/2, f(1 - x) = 1 - f(x). Find f(18/1991).

1991/4. Find a number N with five digits, all different and none zero, which equals the sum of all distinct three digit numbers whose digits are all different and are all digits of N.

1991/5. Let p(m, n) be the polynomial $2m^2 - 6mn + 5n^2$. The range of p is the set of all integers k such that k = p(m, n) for some integers m, n. Find which members of {1, 2, ..., 100} are in the range of p. Show that if h and k are in the range of p, then so is hk.

1991/6. Given three non-collinear points M, N, H show how to construct a triangle which has H as orthocenter and M and N as the midpoints of two sides.

1992/1. a_n is the last digit of 1 + 2 + ... + n. Find $a_1 + a_2 + ... + a_{1992}$.

1992/2. Let $f(x) = a_1/(x + a_1) + a_2/(x + a_2) + ... + a_n/(x + a_n)$, where a_i are unequal positive reals. Find the sum of the lengths of the intervals in which $f(x) \geq 1$.

1992/3. ABC is an equilateral triangle with side 2. Show that any point P on the incircle satisfies $PA^2 + PB^2 + PC^2 = 5$. Show also that the triangle with side lengths PA, PB, PC has area (√3)/4.

1992/4. Let a_n, b_n be two sequences of integers such that: (1) $a_0 = 0$, $b_0 = 8$; (2) $a_{n+2} = 2 a_{n+1} - a_n + 2$, $b_{n+2} = 2 b_{n+1} - b_n$, (3) $a_n^2 + b_n^2$ is a square for n > 0. Find at least two possible values for (a_{1992}, b_{1992}).

1992/5. Construct a cyclic trapezium ABCD with AB parallel to CD, perpendicular distance h between AB and CD, and AB + CD = m.

1992/6. Given a triangle ABC, take A' on the ray BA (on the opposite side of A to B) so that AA' = BC, and take A" on the ray CA (on the opposite side of A to C) so that AA" = BC. Similarly take B', B" on the rays CB, AB respectively with BB' = BB" = CA, and C', C" on

the rays AB, CB. Show that the area of the hexagon A"A'B"B'C"C' is at least 13 times the area of the triangle ABC.

1993/1. A palindrome is a positive integers which is unchanged if you reverse the order of its digits. For example, 23432. If all palindromes are written in increasing order, what possible prime values can the difference between successive palindromes take?

1993/2. Show that any convex polygon of area 1 is contained in some parallelogram of area 2.

1993/3. Find all functions f on the positive integers with positive integer values such that (1) if x < y, then f(x) < f(y), and (2) $f(y\, f(x)) = x^2 f(xy)$.

1993/4. ABC is an equilateral triangle. D is on the side AB and E is on the side AC such that DE touches the incircle. Show that AD/DB + AE/EC = 1.

1993/5. If P and Q are two points in the plane, let m(PQ) be the perpendicular bisector of PQ. S is a finite set of n > 1 points such that: (1) if P and Q belong to S, then some point of m(PQ) belongs to S, (2) if PQ, P'Q', P"Q" are three distinct segments, whose endpoints are all in S, then if there is a point in all of m(PQ), m(P'Q'), m(P"Q") it does not belong to S. What are the possible values of n?

1993/6. We say that two non-negative integers are *related,* ~, if their sum uses only the digits 0 and 1. For example 22 and 79 are related. Let A and B be two infinite sets of non-negative integers such that: (1) if a ~ A and b ~ B, then a and b are related, (2) if c is related to every member of A, then it belongs to B, (3) if c is related to every member of B, then it belongs to A. Show that in one of the sets A, B we can find an infinite number of pairs of consecutive numbers.

1994/1. Show that there is a number 1 < b < 1993 such that if 1994 is written in base b then all its digits are the same. Show that there is no number 1 < b < 1992 such that if 1993 is written in base b then all its digits are the same.

1994/2. ABCD is a cyclic quadrilateral. A circle whose center is on the side AB touches the other three sides. Show that AB = AD + BC. What is the maximum possible area of ABCD in terms of |AB| and |CD|?

1994/3. There is a bulb in each cell of an n x n board. Initially all the bulbs are off. If a bulb is touched, that bulb and all the bulbs in the same row and column change state (those that are on, turn off, and those that are off, turn on). Show that it is possible by touching m bulbs to turn all the bulbs on. What is the minimum possible value of m?

1994/4. ABC is an acute-angled triangle. P is a point inside its circumcircle. The rays AP, BP, CP intersect the circle again at D, E, F. Find P so that DEF is equilateral.

1994/5. n and r are positive integers. Find the smallest k for which we can construct r subsets $A_1, A_2, ... , A_r$ of {0, 1, 2, ... , n-1} each with k elements such that each integer $0 \leq m < n$ can be written as a sum of one element from each of the r subsets.

1994/. Show that given any integer $0 < n \leq 2^{1000000}$ we can find at set S of at most 1100000 positive integers such that S includes 1 and n and every element of S except 1 is a sum of two (possibly equal) smaller elements of S.

1995/1. Find all possible values for the sum of the digits of a square.

1995/2. $n > 1$. Find all solutions in real numbers $x_1, x_2, \dots , x_{n+1}$ all at least 1 such that: (1) $x_1^{1/2} + x_2^{1/3} + x_3^{1/4} + \dots + x_n^{1/(n+1)} = n\, x_{n+1}^{1/2}$; and (2) $(x_1 + x_2 + \dots + x_n)/n = x_{n+1}$.

1995/3. L and L' are two perpendicular lines not in the same plane. AA' is perpendicular to both lines, where A belongs to L and A' belongs to L'. S is the sphere with diameter AA'. For which points P on S can we find points X on L and X' on L' such that XX' touches S at P?

1995/4. ABCD is an n x n board. We call a diagonal row of cells a positive diagonal if it is parallel to AC. How many coins must be placed on an n x n board such that every cell either has a coin or is in the same row, column or positive diagonal as a coin?

1995/5. The incircle of the triangle ABC touches the sides BC, CA, AB at D, E, F respectively. AD meets the circle again at X and AX = XD. BX meets the circle again at Y and CX meets the circle again at Z. Show that EY = FZ.

1995/6. f is a function defined on the positive integers with positive integer values. Use $f^m(n)$ to mean f(f(... f(n) ....)) = n where f is taken m times, so that $f^2(n) = f(f(n))$, for example. Find the largest possible $0 < k < 1$ such that for some function f, we have
$f^m(n) \neq n$ for m = 1, 2, ... , [kn], but $f^m(n) = n$ for some m (which may depend on n).

1996/1. Find the smallest positive integer n so that a cube with side n can be divided into 1996 cubes each with side a positive integer.

1996/2. M is the midpoint of the median AD of the triangle ABC. The ray BM meets AC at N. Show that AB is tangent to the circumcircle of NBC iff $BM/MN = (BC/BN)^2$.

1996/3. $n = k^2 - k + 1$, where k is a prime plus one. Show that we can color some squares of an n x n board black so that each row and column has exactly k black squares, but there is no rectangle with sides parallel to the sides of the board which has its four corner squares black

1996/4. $n > 2$ is an integer. Consider the pairs (a, b) of relatively prime positive integers, such that $a < b \leq n$ and $a + b > n$. Show that the sum of 1/ab taken over all such pairs is 1/2.

1996/5. An equilateral triangle of side n is divided into n^2 equilateral triangles of side 1 by lines parallel to the sides. Initially, all the sides of all the small triangles are painted blue. Three coins A, B, C are placed at vertices of the small triangles. Each coin in turn is moved a distance 1 along a blue side to an adjacent vertex. The side it moves along is painted red, so once a coin has moved along a side, the side cannot be used again. More than one coin is allowed to occupy the same vertex. The coins are moved repeatedly in the order A, B, C, A, B, C, ... Show that it is possible to paint all the sides red in this way.

1996/6. $A_1, A_2, \dots , A_n$ are points in the plane. A non-zero real number k_i is assigned to each point, so that the square of the distance between A_i and A_j (for $i \neq j$) is $k_i + k_j$. Show that n is at most 4 and that if n = 4, then $1/k_1 + 1/k_2 + 1/k_3 + 1/k_4 = 0$.

1997/1. $k \geq 1$ is a real number such that if m is a multiple of n, then [mk] is a multiple of [nk]. Show that k is an integer.

1997/2. I is the incenter of the triangle ABC. A circle with center I meets the side BC at D and P, with D nearer to B. Similarly, it meets the side CA at E and Q, with E nearer to C, and it meets AB at F and R, with F nearer to A. The lines EF and QR meet at S, the lines FD and RP meet at T, and the lines DE and PQ meet at U. Show that the circumcircles of DUP, ESQ and FTR have a single point in common.

1997/3. $n > 1$ is an integer. D_n is the set of lattice points (x, y) with $|x|, |y| \leq n$. If the points of D_n are colored with three colors (one for each point), show that there are always two points with the same color such that the line containing them does not contain any other points of D_n. Show that it is possible to color the points of D_n with four colors (one for each point) so that if any line contains just two points of D_n then those two points have different colors.

1997/4. Let o(n) be the number of 2n-tuples $(a_1, a_2, \dots , a_n, b_1, b_2, \dots , b_n)$ such that each $a_i, b_j = 0$ or 1 and $a_1b_1 + a_2b_2 + \dots + a_nb_n$ is odd. Similarly, let e(n) be the number for which the sum is even. Show that $o(n)/e(n) = (2^n - 1)/(2^n + 1)$.

1997/5. ABC is an acute-angled triangle with orthocenter H. AE and BF are altitudes. AE is reflected in the angle bisector of angle A and BF is reflected in the angle bisector of angle B. The two reflections intersect at O. The rays AE and AO meet the circumcircle of ABC at M and N respectively. P is the intersection of BC and HN, R is the intersection of BC and OM, and S is the intersection of HR and OP. Show that AHSO is a parallelogram.

1997/6. Given 1997 points inside a circle of radius 1, one of them the center of the circle. For each point take the distance to the closest (distinct) point. Show that the sum of the squares of the resulting distances is at most 9.

1998/1. There are 98 points on a circle. Two players play alternately as follows. Each player joins two points which are not already joined. The game ends when every point has been joined to at least one other. The winner is the last player to play. Does the first or second player have a winning strategy?

1998/2. The incircle of the triangle ABC touches BC, CA, AB at D, E, F respectively. AD meets the circle again at Q. Show that the line EQ passes through the midpoint of AF iff AC = BC.

1998/3. Find the smallest number n such that given any n distinct numbers from {1, 2, 3, ... , 999}, one can choose four different numbers a, b, c, d such that a + 2b + 3c = d.

1998/4. Representatives from $n > 1$ different countries sit around a table. If two people are from the same country then their respective right hand neighbors are from different countries. Find the maximum number of people who can sit at the table for each n.

1998/5. $P_1, P_2, \ldots, P_n$ are points in the plane and $r_1, r_2, \ldots, r_n$ are real numbers such that the distance between P_i and P_j is $r_i + r_j$ (for i not equal to j). Find the largest n for which this is possible.

1998/6. k is the positive root of the equation $x^2 - 1998x - 1 = 0$. Define the sequence $x_0, x_1, x_2, \ldots$ by $x_0 = 1$, $x_{n+1} = [k\, x_n]$. Find the remainder when x_{1998} is divided by 1998.

1999/1. Find all positive integers $n < 1000$ such that the cube of the sum of the digits of n equals n^2.

1999/2. Given two circles C and C' we say that C *bisects* C' if their common chord is a diameter of C'. Show that for any two circles which are not concentric, there are infinitely many circles which bisect them both. Find the locus of the centers of the bisecting circles.

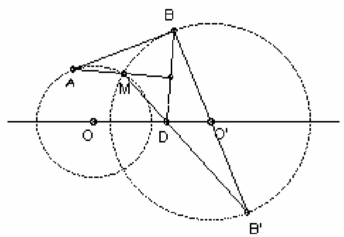
1999/3. Given points $P_1, P_2, \ldots, P_n$ on a line we construct a circle on diameter P_iP_j for each pair i, j and we color the circle with one of k colors. For each k, find all n for which we can always find two circles of the same color with a common external tangent.

1999/4. Show that any integer greater than 10 whose digits are all members of {1, 3, 7, 9} has a prime factor ≥ 11.

1999/5. O is the circumcenter of the acute-angled triangle ABC. The altitudes are AD, BE and CF. The line EF cuts the circumcircle at P and Q. Show that OA is perpendicular to PQ. If M is the midpoint of BC, show that $AP^2 = 2\, AD\cdot OM$.

1999/6. Given two points A and B, take C on the perpendicular bisector of AB. Define the sequence $C_1, C_2, C_3, \ldots$ as follows. $C_1 = C$. If C_n is not on AB, then C_{n+1} is the circumcenter of the triangle ABC_n. If C_n lies on AB, then C_{n+1} is not defined and the sequence terminates. Find all points C such that the sequence is periodic from some point on.

2000/1. Label the vertices of a regular n-gon from 1 to $n > 3$. Draw all the diagonals. Show that if n is odd then we can label each side and diagonal with a number from 1 to n different from the labels of its endpoints so that at each vertex the sides and diagonals all have different labels.



2000/2. Two circles C and C' have centers O and O' and meet at M and N. The common tangent closer to M touches C at A and C' at B. The line through B perpendicular to AM meets the line OO' at D. BO'B' is a diameter of C'. Show that M, D and B' are collinear.

2000/3. Find all solutions to $(m + 1)^a = m^b + 1$ in integers greater than 1.

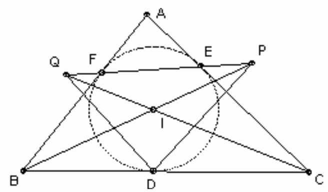
2000/4. Some terms are deleted from an infinite arithmetic progression 1, x, y, ... of real numbers to leave an infinite geometric progression 1, a, b, ... Find all possible values of a.

2000/5. Given a pile of 2000 stones, two players take turns in taking stones from the pile. Each player must remove 1, 2, 3, 4, or 5 stones from the pile at each turn, but may not take the same number as his opponent took on his last move. The player who takes the last stone wins. Does the first or second player have a winning strategy?

2000/6. A convex hexagon is called a *unit* if it has four diagonals of length 1, whose endpoints include all the vertices of the hexagon. Show that there is a unit of area k for any $0 < k \leq 1$. What is the largest possible area for a unit?

2001/1. Show that there are arbitrarily large numbers n such that: (1) all its digits are 2 or more; and (2) the product of any four of its digits divides n.

2001/2. ABC is a triangle. The incircle has center I and touches the sides BC, CA, AB at D, E, F respectively. The rays BI and CI meet the line EF at P and Q respectively. Show that if DPQ is isosceles, then ABC is isosceles.



2001/3. Let X be a set with n elements. Given $k > 2$ subsets of X, each with at least r elements, show that we can always find two of them whose intersection has at least $r - nk/(4k - 4)$ elements.

2001/4. Call a set of 3 distinct elements which are in arithmetic progression a *trio*. What is the largest number of trios that can be subsets of a set of n distinct real numbers?

2001/5. Two players play a game on a 2000 x 2001 board. Each has one piece and the players move their pieces alternately. A *short move* is one square in any direction (including diagonally) or no move at all. On his first turn each player makes a short move. On subsequent turns a player must make the same move as on his previous turn followed by a short move. This is treated as a single move. The board is assumed to wrap in both directions so a player on the edge of the board can move to the opposite edge. The first player wins if he can move his piece onto the same square as his opponent's piece. For example, suppose we label the squares from (0, 0) to (1999, 2000), and the first player's piece is initially at (0, 0) and the second player's at (1996, 3). The first player could move to (1999, 2000), then the second player to (1996, 2). Then the first player could move to (1998, 1998), then the second player to (1995, 1). Can the first player always win irrespective of the initial positions of the two pieces?

2001/6. Show that a square with side 1 cannot be covered by five squares with side less than 1/2.

2002/1. The numbers 1, 2, ... , 2002 are written in order on a blackboard. Then the 1st, 4th, 7th, ... , 3k+1th, ... numbers in the list are erased. Then the 1st, 4th, 7th, ... 3k+1th numbers in the remaining list are erased (leaving 3, 5, 8, 9, 12, ...). This process is carried out repeatedly until there are no numbers left. What is the last number to be erased?

2002/2. Given a set of 9 points in the plane, no three collinear, show that for each point P in the set, the number of triangles containing P formed from the other 8 points in the set must be even.

2002/3. ABC is an equilateral triangle. P is a variable interior point such that angle APC = 120^o. The ray CP meets AB at M, and the ray AP meets BC at N. What is the locus of the circumcenter of the triangle MBN as P varies?

2002/4. ABC is a triangle. BD is the an angle bisector. E, F are the feet of the perpendiculars from A, C respectively to the line BD. M is the foot of the perpendicular from D to the line BC. Show that ∢DME = ∢DMF.

2002/5. The sequence a_n is defined as follows: $a_1 = 56$, $a_{n+1} = a_n - 1/a_n$. Show that $a_n < 0$ for some n such that $0 < n < 2002$.

2002/6. A game is played on a 2001 x 2001 board as follows. The first player's piece is the policeman, the second player's piece is the robber. Each piece can move one square south, one square east or one square northwest. In addition, the policeman (but not the robber) can move from the bottom right to the top left square in a single move. The policeman starts in the central square, and the robber starts one square diagonally northeast of the policeman. If the policeman moves onto the same square as the robber, then the robber is captured and the first player wins. However, the robber may move onto the same square as the policeman without being captured (and play continues). Show that the robber can avoid capture for at least 10000 moves, but that the policeman can ultimately capture the robber.

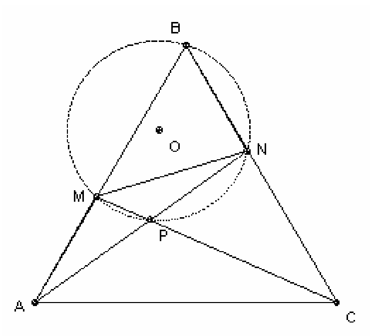


Figura **A3**

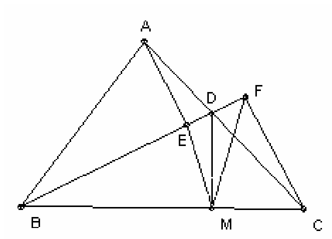


Figura **B1**

2003/1. Let A, B be two sets of N consecutive integers. If N = 2003, can we form N pairs (a, b) with a ∈ A, b ∈ B such that the sums of the pairs are N consecutive integers? What about N = 2004?

2003/2. C is a point on the semicircle with diameter AB. D is a point on the arc BC. M, P, N are the midpoints of AC, CD and BD. The circumcenters of ACP and BDP are O, O'. Show that MN and OO' are parallel.

2003/3. Pablo was trying to solve the following problem: find the sequence $x_0, x_1, x_2, \ldots, x_{2003}$ which satisfies $x_0 = 1$, $0 \le x_i \le 2 x_{i-1}$ for $1 \le i \le 2003$ and which maximises S. Unfortunately he could not remember the expression for S, but he knew that it had the form $S = \pm x_1 \pm x_2 \pm \ldots \pm x_{2002} + x_{2003}$. Show that he can still solve the problem.

2003/4. A = {1, 2, 3, ... , 49} does not contain six consecutive integers. Find the largest possible value of |A|. How many such subsets are there (of the maximum size)?

2003/5. ABCD is a square. P, Q are points on the sides BC, CD respectively, distinct from the endpoints such that BP = CQ. X, Y are points on AP, AQ respectively. Show that there is a triangle with side lengths BX, XY, YD.

2003/6. The sequences $a_0, a_1, a_2, \ldots$ and $b_0, b_1, b_2, \ldots$ are defined by $a_0 = 1$, $b_0 = 4$, $a_{n+1} = a_n^{2001} + b_n$, $b_{n+1} = b_n^{2001} + a_n$. Show that no member of either sequence is divisible by 2003.

2004/1. Deve-se colorir as casas de um tabuleiro de 1001×1001 de acordo com as seguintes regras:

- o Se duas casas têm um lado comum, então pelo menos uma delas deve ser colorida.
- o De cada seis casas consecutivas de uma linha ou de uma coluna, devem colorir-se sempre pelo menos duas delas que sejam adjacentes.

Determinar o número mínimo de casas que devem ser coloridas.

2004/2. Se considera no plano uma circunferência de centro *O* e raio *r*, e um ponto *A* exterior a ela. Seja *M* um ponto da circunferência e *N* o ponto diametralmente oposto a *M*. Determinar o lugar geométrico dos centros das circunferências que passam por *A*, *M* e *N* quando *M* varia.

2004/3. Sejam *n* e *k* números inteiros positivos tais que *n* é ímpar ou *n* e *k* são pares. Provar que existem inteiros *a* e *b* tais que

$$\text{mdc}(a, n) = \text{mdc}(b, n) = 1 \quad \text{e} \quad k = a + b.$$

2004/4. Determinar todos os pares (a,b), onde *a* e *b* são números inteiros positivos de dois dígitos cada um, tais que $100a + b$ e $201a + b$ são quadrados perfeitos de quatro dígitos.

2004/5. Dado um triângulo escaleno *ABC*, se designam por *A′, B′* e *C′* os pontos de intersecção das bissectrizes interiores dos ângulos *A, B* e *C* com os lados opostos, respectivamente.
Sejam:

A″ a intersecção de *BC* com a mediatriz de *AA′*,
B″ a intersecção de *AC* com a mediatriz de *BB′* e

C'' a intersecção de *AB* com a mediatriz de *CC'*.
Provar que *A''*, *B''* e *C''* são colineares.

2004/6. Para um conjunto H de pontos no plano, diz-se que um ponto *P* do plano é um *ponto de corte* de H, se existem quatro pontos distintos *A, B, C* e *D* em H tais que as rectas *AB* e *CD* são distintas e se cortam em *P*.
Dado um conjunto finito A_0 de pontos no plano, se constrói uma sucessão de conjuntos A_1, A_2, A_3, … da seguinte forma: para qualquer $j \geq 0$, A_{j+1} é a união de A_j com o conjunto de todos os pontos de corte de A_j.
Demonstrar que se a união de todos os conjuntos da sucessão é um conjunto finito, então para qualquer $j \geq 1$ tem-se $A_j = A_1$.
